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Why Do We Need a Disequilibrium Model of European Open Professional Team Sports Leagues?

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When substituting win-maximising to profit-maximising as the club's objective function in European open leagues, model builders did not mind that managing a club without profit-maximising boils down to accepting that the clubs budget constraint usually will be soft (SBC).
With SBC an equilibrium model turns into a disequilibrium model.

1. The standard model of a European open team sports league

Elaborated on by Sloane, Késenne, Szymanski, the model ended up as:

$$Max \;\; t_i \qquad (1)$$

$$R_i\,(m_i\,,\,t_i\,) - \;\; s.\; t_i - c_i^{\,0} = 0 \qquad (2)$$

t_i the quantity of talent in a club i, R_i the club i's revenue function, m_i the size of its local market, s the market unit cost of talent and $c_i^{\,0}$ a fixed cost

For the model resolution, see the full text and literature (Késenne, 2007), namely the solution: $R_i - s.\ t_i = 0$ (5).

2. Some less heeded and debatable assumptions of the standard model

To allow writing a team's balanced budget constraint, it is needed to have:
1/ infinite price (wage) and quantity of talent flexibility;
2/ identical units of homogenous talent, additive and substitutive;
3/ clubs assumed to be managed in such a way as to break even.

A comprehensive model of a team sports leagues must integrate equations describing a second (output) market, not only the (input) talent market. Rare, an exception is Késenne (2007).

Assumption 2 must definitely be dropped since:
a/ rather abstract concept of a homogenous unit of talent: impossible to compare how many Bosman in one Messi (dead end for empirical testing);

b/ the terms for trading superstars are dramatically different than for day men players (not even a market homogeneity of talent/labour);
c/ there exists a whole theory of labour market segmentation (can be applied here, see Bourg, 1983: market for French soccer players, Vrooman, 1996: American baseball players);
d/ various theories of skyrocketing wages of superstars: here we assume that they are in monopoly (situation and rent) on the supply side of the talent market.

The demand for games (attendances) often tested as being inelastic to price; prices are not determined according to quantity (spectators) but beforehand, at the dawn of the season or before a game = rigid or sticky prices.

A balanced budget constraint introduces a heavy/unrealistic assumption since being in the red is more the rule (56% clubs UEFA) than exception in European soccer, and a number of clubs are bailed out.

3. A simple disequilibrium model: price rigidity on the labour market for talent

It writes:

$$Max \;\; t_i \qquad (6')$$

$$R_i\,(m_i\,,\,t_i\,) - \;\; s.\; t_i - c_i^{\,0} \;\; \leq \textbf{or} \geq 0 \qquad (7')$$

Skipping out the constant cost, if stadium and management are not major inputs of a club's production and revenue function, it comes:

$$R_i\,(m_i\,,\,t_i\,) \leq \textbf{or} \geq s.\; t_i \qquad (8')$$

Due to wage rigidity and thus limited talent mobility, equilibrium in (5) cannot be reached: neither equilibrium quantity of talent, nor equilibrium wage.

The constraint (7') may be either ≤ 0 (excess demand for talent) or ≥ 0 (too short demand or excess supply). The model is unspecified: no endogenous variable determines which disequilibrium will prevail.

Possible exogenous variables from outside the model (ex: strong players' union = downward wage stickiness; strong owners = wage sticky upwards).

If a second market for sport shows, in a disequilibrium model, it may be in excess demand or excess supply as well (endogenously unspecified) so that three disequilibrium regimes can emerge (Benassy, 1982), see Table 1:

. Double excess demand: repressed inflation
. Product excess demand/labour excess supply: classical unemployment
. Double excess supply: Keynesian unemployment

A candidate variable to specify the model (beyond price rigidity) is a soft budget constraint that allows the club to be in the red, then (7') is clearly $\leq$, *i.e.* an excess demand for talent.

4. Disequilibrium economics and soft budget constraint (SBC)

A disequilibrium model is quite more general than an equilibrium model:
. Infinite number of solutions such as ≤ 0 (or ≥ 0) vs. a unique solution ($= 0$).
. Though more stylish, elegant and comfortable to build up an equilibrium model.
. A unique solution (equilibrium) does not often show up in actual markets.
. Most actual (sports) markets reproduce themselves in disequilibria (stored, stock-pilled goods; or queuing and waiting lists).
. Practically, a market which would have reached perfect equilibrium would be empty of products on sale (then all traders back home), sellers and buyers.

Kornaï's SBC economics applies to planned economies and various industries in market economies, when:

1/ Firms (clubs) are price makers on input and output markets; if price takers, then hard budget constraint (HBC);

2/ Firms can avoid taxation or influence taxation rules (tax arrears), if not HBC.
3/ Firms can receive state/municipality grants and subsidies (subsidised rate in renting a stadium, *salve Calcio*), if not HBC.
4/ Firms can be granted credit by banks (Spanish banks), investment funds, etc., whatever their deficit, debt and insolvency, if not HBC.
5/ Firms' investments are not dependent on retained profits because they can find external investment finance (banks, sugar daddies), if not HBC.
When SBC, a club/firm spends more than its revenues and overinvest (in talent).

5. European soccer clubs' soft budget constraint: empirical evidence

Storm & Nielsen (2012) develop a number of arguments (some already sketched in Andreff 2007) proving the existence of SBC in European football leagues: clubs recurring losses, too big to fail (to their stakeholders) always find someone to bail them out (soft subsidies, sugar daddies, sponsors), fail to pay (taxes, arrears, etc.), growing debts … and survive in the business!
Many more examples in the literature, ex: *JSE* special issues 1/2006 & 6/2007.

6. A labour market for players talent in excess demand

Now specified by the SBC (7):

$$Max \ \ t_i \qquad (6)$$

$$R_i\,(m_i\,,\,t_i\,) - \ s.\,t_i - c_i^{\,0} \ \leq 0 \qquad (7)$$

$$R_i\,(m_i\,,\,t_i\,) \leq \ s.\,t_i \qquad (8)$$

If all (most) clubs behave this way: the league itself is in the red and there is a permanent excess demand on the labour market for talent (the arm's race) given the existing (non infinite) number of talented players: T_0

Therefore (9) : $$\sum_{i=1}^{n} t_i \ = \ T \ \ ; \ T \ \geq \ T_0$$

shows a labour market for talents in excess demand in the face of a limited supply.

With an excess demand for talent, the marginal revenue productivity of labour cannot equalise the marginal unit cost of labour (wage), so that:

$$RM_i = \frac{\partial R_i(m_i, t_i)}{\partial t_i} \leq s \quad (10)$$

The last units of labour are overpaid in terms of wages with excess demand disequilibrium. Due to overall excess demand, the clubs are rationed by a too short supply of talent and are eager to pay a salary much higher than the marginal labour productivity of talent (they overpay players in order to attract them).

The last players recruited provide a lower labour productivity than the salary they are paid for, which sounds like the exact contrary of Scully's sense (1974) of players' exploitation. Clubs are embarked on an arm's race to recruit more players (rosters have to be limited) and are involved in an endless skyrocketing trend of payroll increase.

A salary cap would not be a first step on the path toward moderating payroll growth as long as the labour market were to be in excess demand (*i.e.* the clubs' budget constraints were not hardened in such a way as tolerating no club deficit).

7. Introducing a market for sport shows in short supply

Let us first assume no difference between a club's fans and (couch potato) TV viewers. Demand side = fans only demanding their favourite team wins and then attend as many games as they can financially afford.

The league adopts the usual monopoly strategy of shortening supply in view of raising price and maximise its monopoly rent: total supply of games is $n\ (n-1)$ if n teams in the league (no consideration here of stadium capacity).

Each fan of a team i maximises the utility of his/her games consumption Cgi which grows with the number of attended games ni :

$$Max\ Cgi\ (\ ni\ ) \quad (11)$$

$$ni\ (\ m0i,\ \ Inci\ ,\ \underline{p}\ ) \geq\ 2\ (n-1)\ \underline{p} \quad (12)$$

with the fan's initial money endowment $m0i$ (savings, property), his/her current income share available for paying tickets at the gate $Inci$, and the fixed price $\underline{p}$ of a game. The fan's expenditure over a season cannot be higher than $2\ (n-1)\ \underline{p}$, and (12) expresses his/her excess demand.

A fixed/sticky price immediately generates market disequilibrium (Benassy). Many studies have concluded to a demand for games inelastic to price, thus the price variable can be dropped and (12) simplifies:

$$ni\ (\ m0i\ ,\ \ Inci\ \ ) \geq 2\ (n-1)\ (13)$$

At the league level, that is for all games and all fans, market disequilibrium is a *short supply* (shortage) of games compared to fan's demand:

$$\sum_{i=1}^{n} n_i\ (m_{0i}\ , Inc_i\ ) \geq 2n(n-1)\ \ (14)$$

Note: no variable expressing a game quality (competitive balance, win percentages) is needed because it is assumed that a true team *i*'s fan is not attracted by game quality or even his/her favourite team's quality (emotional determinants to attend under a budget constraint *Inci*).

. From (6) to (14) = a model of a European football league with clubs' SBC operating under a ***repressed inflation*** regime or a ***shortage*** economy or industry (Kornaï):

. No chance to reach economic equilibrium without phasing out the SBC.

. UEFA financial fair play (to be assessed in this perspective) probably would not (€5 and 45 million deficit allowances, *i.e.* bail outs) stop the SBC and transform it into a HBC; it will only harden it to some extent.

8. Dropping the assumption of a homogenous unit of talent: labour market segmentation

Let us now drop the assumption of a homogenous unit of talent.

The aforementioned excess demand relationship (9) pertains only to (highly talented) superstars, not to day men/average players (often unemployed = excess supply that contradicts inequality 9 above).

Labour market segmentation is deeply rooted in this differentiation between superstars in excess demand and day men players in excess supply: a superstar stands in a monopoly position (over his rare talent) on the supply side, a monopoly rent is included in his wage (he cannot be substituted by another player).

We assume that excess demand for superstars is not compensated by the excess supply for average players so that (9) is still relevant for the market overall and writes as (18) below.

Let *Ts* stand for the number of superstars and *Ta* the number of day men/average players available in the league, then:

$$Ts + Ta = T0 \quad (15)$$

Now a club has to maximise an assortment of superstars and day men/average players in order to maximise its wins (16) under a SBC (17):

$$Max\ (tsi + tai\ ) \quad (16)$$

$$Ri\ (mi\ ,\ tsi\ + tai\ ) \leq\ ss\ .\ tsi + sa\ .\ tai \quad (17).$$

and:

$$\sum_{i=1}^{n} (t_{si} + t_{ai}) \geq T_0 \qquad (18)$$

once t_{si} is defined as the demand of superstars by team *i*, t_{ai} as the demand of average players by team *i*, s_s as the market wage of superstars, and s_a as the market wage of average players

Excess demand in the market segment for superstars writes as (19) and coincides with a wage higher than revenue marginal productivity of labour (20):

$$\sum_{i=1}^{n} t_{si} \geq T_s \qquad (19)$$

$$RM_{si} = \frac{\partial R_i(m_i, t_{si})}{\partial t_i} \leq s_s \qquad (20)$$

while in the day men/average player segment of the market an excess supply of talents (21) drives the market wage to be lower than marginal labour productivity:

$$\sum_{i=1}^{n} t_{ai} \leq T_a \qquad (21)$$

$$RM_{ai} = \frac{\partial R_i(m_i, t_{ai})}{\partial t_i} \geq s_a \qquad (22)$$

Inequality (22) shows that average players are subject to exploitation in a Scully's sense, they are paid less than their marginal productivity suffering from being in excess supply as well as from the monopsonistic position of the league in the labour market (and the rent levied by the league on average players' salaries).

A disequilibrium model of a league with win-maximising clubs operating under a SBC in a segmented labour market explains by the same token the arm's race for superstars, the superstars skyrocketing wages triggering clubs payrolls overruns, the unemployment of average players, and their low wages (below their marginal labour productivity).

9. Differentiating between fans and TV viewers then between free-to-air and pay-per-view TV channels: a differentiated product market

Buraimo, Simmons, others distinguish the fans' demand for games on the pitch from a TV viewers' demand for televised games – on the screens (two sport shows derived from a same game). Do not behave the same way.

TV viewers demand interesting/balanced/quality games with a high outcome uncertainty. Thus two segments in the product market of a sport league.
The fans segment is described by above inequalities (11) to (14) – unchanged.

If TV viewer audience is attracted by outcome uncertainty, the utility of their consumption is a function of the competitive balance - win percentages *w*.

Over some threshold of the couple (*wi* , *wj*) a game is considered as attractive (high quality product). Beyond the threshold they switch their TV on and their demand of high quality games is big while only a proportion $\lambda < 1$ of overall games supplied by the league each season pass the quality threshold. Then:

$$Max\ C'gi\ (\ wi\ ,\ wj\ )\quad (23)$$

$$n'i\ (\ m'0i,\ \ Inc'i\ \ ) \geq \lambda\ (n-1)\quad (24)$$

$$p = 0 \qquad (25)$$

assuming that games are TV broadcast for free on free-to-air channels, and *C'gi* , *n'i* , *m'0i* , *Inc'i* symbols have the same meaning as in (11) and (12), but for TV viewers instead of fans.

Since high quality games are only a share of all the games broadcast after the contract signed between the league and one TV channel, TV viewers are rationed in high quality games (monopoly TV channel or collusive oligopoly). The market is in excess demand (26) for televised high quality games:

Low or close to zero audience is reached for a number of boring or without outcome uncertainty games simply because: $\lambda \, . \, n\,(n-1) \; < \; n\,(n-1)$.

$$\sum_{i=1}^{n} n'_i \,(m'_i \,, Inc'_i \,) \geq \lambda \; n \; (n-1) \qquad (26)$$

Now let us consider that televised games are no longer broadcast by free-to-air but pay-per-view and commercial TV channels at a subscription (fee) price p^*. The market equilibrium or disequilibrium will crucially depend on the level at which the subscription fee is fixed by TV companies in monopoly or collusive oligopoly. TV viewers will take into account p^* in their utility-maximising behaviour so that:

$$Max\ C'g_i\ (w_i\ ,\ w_j\ ,\ p^*) \quad (27)$$

$$n'_i\ (m'_{0i},\ \ Inc'_i\ ,\ p^*) \geq \textbf{or} \leq \lambda\ (n-1)\ p^* \quad (28)$$

$$p^* > \ 0 \qquad (29)$$

If by chance the TV channel which signs TV rights with a league at the dawn of the football season exactly finds ex ante the single value which transforms the (28) inequality into an equality, then the TV company has found the price p^* that TV viewers are willing to pay for a proportion λ of all the broadcast games without spending more than they can afford: p^* is the equilibrium price.

In current functioning of a Euroepan football league and bargaining with overbidding TV channels, this equilibrium subscription fee can actually be found only by chance. If the TV channel fixes the subscription fee lower than p^*, (28) will be $\geq$ and TV viewers will be rationed in high quality games but will pay a low subscription fee for this ration. If, as it is more likely to happen with monopolistic/oligopolistic commercial TV companies, the subscription fee is fixed higher than p^*, a number of potential subscribers will not subscribe, (28) will be $\leq$ and the market will be in excess supply.

Empirical evidence shows signs of excess supply in the market segment of non-free televised sport show (French example below).

France 2010: 98,000 hours of sport televised by pay-per-view and commercial companies against 2,000 hours by free-to-air channels; all subscription fees paid together would have cost over € 2,000 per year to a subscriber willing to pay for all the broadcast sports. If some addicted TV viewers can envisage spending such an amount for watching sports all day and all year long, nobody can spend 268 hours per day (98,000 / 365) in the face of his/her TV screen.

In a next step, a third market should be introduced in the model, *i.e.* the market for TV broadcasting rights on which a football league supplies its games to TV channels against TV rights revenues while TV companies demand and pay the rights to broadcast the league's games. This will help determining p^* since broadcasting rights are a basic share in the production cost of a TV channel.

Conclusion:

In such model, the issue of competitive balance or imbalance will depend on the respective magnitude of financial losses and gains borne by the different clubs in a same league and on the respective share of fans compared to TV viewers (avenues for further research).