Wage dispersion and team performance :
a theoretical model and a semi-parametric analysis

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Context

- What is relevant to explain the economic agent performances is not only their absolute welfare but also their relative situation
- The intra-team wage distribution is thus of great interest to explain individual effort and so collective performance
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  - The tournament theory (Lazear and Rosen 1981) or the danger-potential hypothesis (Ramaswamy and Rowthorn 1991) in favor of a hierarchical structure of wage distribution
  - The pay compression or the inequality aversion hypotheses in favor of a less concentrated distribution (Levine (1991) or Fehr and Schmidt (1999)

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  - Very few efforts have been made on the investigation of non linearities (Franck and Nuesch, 2007)

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In which way the wage dispersion affects the team performance? Which hypothesis holds?

The contest

- Consider a contest between two teams, labeled team A and team B
- Let e^A and e^B represent the efforts of team A's players and team B's players in the game

$$\pi^A(e^A, e^B) = \frac{e^A}{e^A + e^B} \tag{1}$$

$$\pi^B(e^A, e^B) = 1 - \pi^A(e^A, e^B)$$

The payoff

- Each team is composed of n players
- w^A is the total wage bill of team A
- The wage of player p in team A, $p = 1 \dots n$, is w_p^A
- The total effort of team A is $\sum_{p=1}^{n} e_p^A = e^A$
- Player p in team A chooses its effort level e_p^A so as to maximize its payoff

$$U_p^A(e_p^A) = \frac{e^A}{e^A + e^B} - \left(\frac{e_p^A}{w_p^A}\right)^\lambda, p = 1 \dots n \quad (2)$$

with $\lambda > 0$

Team reaction functions

- The effort level of player p in team A must satisfy the following first-order condition

$$\frac{\partial U_p^A}{\partial e_p^A} = \frac{e^B}{(e^A + e^B)^2} - \frac{\lambda}{w_p^A}\left(\frac{e_p^A}{w_p^A}\right)^{\lambda-1} = 0, p = 1 \ldots n \quad (3)$$

- Rewriting the first-order condition gives

$$e^A = \sum_{p=1}^{n} (w_p^A)^{\frac{\lambda}{\lambda-1}} \left(\frac{e^B}{\lambda(e^A + e^B)^2}\right)^{\frac{1}{\lambda-1}} \quad (4)$$

- which implicitly defines the optimal response $e^A = e^A(e^B)$ of team A to any effort level e^B chosen by team B

Team reaction functions

- By symmetry, we find that

$$e^B = \sum_{p=1}^{n} (w_p^B)^{\frac{\lambda}{\lambda-1}} \left(\frac{e^A}{\lambda(e^A + e^B)^2} \right)^{\frac{1}{\lambda-1}} \quad (5)$$

- Which implicitely defines the reaction function of team B, that is, $e^B = e^B(e^A)$
- It follows from Equations 4 and 5 that

$$\frac{e^A}{e^B} = \left(\frac{X^A}{X^B} \right)^{\frac{\lambda-1}{\lambda}}, \quad (6)$$

with $X^A = \sum_{p=1}^{n} \left(w_p^A\right)^{\frac{\lambda}{\lambda-1}}$ and $X^B = \sum_{p=1}^{n} \left(w_p^B\right)^{\frac{\lambda}{\lambda-1}}$

- The functions X^A, may be re-written as
  $X^A = \left(w^A\right)^{\frac{\lambda}{\lambda-1}} \sum_{p=1}^{n} \left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}}$,

  where $\alpha_p^A = \frac{w_p^A}{w^A}$ is the share of total wage held by player p in team A
- The term $\sum_{p=1}^{n} \left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}}$ is a measure of the concentration of wages in team A
- For a given w^A, the function X^A increases (resp. decreases) whith the concentration of team A's wages when $\lambda > 1$ (resp. $\lambda < 1$).

Equilibrium

- Using Equation 6 with Equations 4 and 5 yields the Nash equilibrium effort levels of team A and team B :

$$e^{A*} = \left(X^A\right)^{\frac{\lambda-1}{\lambda}} \left(\frac{\left(X^A\right)^{\frac{\lambda-1}{\lambda}} \left(X^B\right)^{\frac{\lambda-1}{\lambda}}}{\lambda \left(\left(X^A\right)^{\frac{\lambda-1}{\lambda}} + \left(X^B\right)^{\frac{\lambda-1}{\lambda}} \right)^2} \right)^{\frac{1}{\lambda}} \quad (7)$$

$$e^{B*} = \left(X^B\right)^{\frac{\lambda-1}{\lambda}} \left(\frac{\left(X^A\right)^{\frac{\lambda-1}{\lambda}} \left(X^B\right)^{\frac{\lambda-1}{\lambda}}}{\lambda \left(\left(X^A\right)^{\frac{\lambda-1}{\lambda}} + \left(X^B\right)^{\frac{\lambda-1}{\lambda}} \right)^2} \right)^{\frac{1}{\lambda}} \quad (8)$$

Equilibrium

- By using Equation 6 with Equation 1, find the equilibrium values of the contest success functions

$$\pi^A(e^{A*}, e^{B*}) = \frac{1}{1 + \left(\frac{X^B}{X^A}\right)^{\frac{\lambda-1}{\lambda}}} = \frac{1}{1 + \frac{w^B}{w^A}\left(\frac{\sum_{p=1}^{n}\left(\alpha_p^B\right)^{\frac{\lambda}{\lambda-1}}}{\sum_{p=1}^{n}\left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}}}\right)^{\frac{\lambda-1}{\lambda}}} \quad (9)$$

$$\pi^B(e^{A*}, e^{B*}) = \frac{1}{1 + \left(\frac{X^A}{X^B}\right)^{\frac{\lambda-1}{\lambda}}} = \frac{1}{1 + \frac{w^A}{w^B}\left(\frac{\sum_{p=1}^{n}\left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}}}{\sum_{p=1}^{n}\left(\alpha_p^B\right)^{\frac{\lambda}{\lambda-1}}}\right)^{\frac{\lambda-1}{\lambda}}} \quad (10)$$

Proposition

- Team A probability to win (π^A) is:
  - an increasing function of its relative wage bill $\frac{w^A}{w^B}$
  - a decreasing (resp. increasing) function of its relative wage concentration $\left(\frac{\sum_{p=1}^{n} \left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}}}{\sum_{p=1}^{n} \left(\alpha_p^B\right)^{\frac{\lambda}{\lambda-1}}} \right)^{\frac{\lambda-1}{\lambda}}$ if $\lambda < 1$ (resp. if $\lambda > 1$)
- If $\lambda < 1$ the pay compression hypothesis holds
- If $\lambda > 1$ the tournament theory holds

Major League Baseball matches

Why?

- The data exist and are available
- A match is a natural experiment (controlled environment, all matches with the same rules set, individual efforts and wages are observable...)
- Baseball features (no draw, stability...)

(Match level data $\approx$ 10000 observations, regular season...)

Lambda estimation

From

$$\pi^A = \frac{1}{1 + \frac{w^B}{w^A}\left(\frac{\sum_{p=1}^{n}\left(\alpha_p^B\right)^{\frac{\lambda}{\lambda-1}}}{\sum_{p=1}^{n}\left(\alpha_p^A\right)^{\frac{\lambda}{\lambda-1}}}\right)^{\frac{\lambda-1}{\lambda}}}$$

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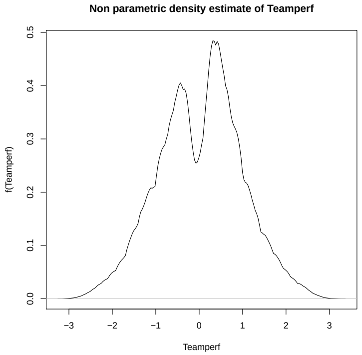
- By Max. Likelihood, we find $\lambda = 0.34$
  the wage concentration harms the team performance

Results

- The proposed theoretical framework is general enough to encompass both competing theories
- We found $\lambda = 0.34$ by Max. Likelihood,
  it pleads in favor of the pay compression hypothesis rather than the tournament theory
- The reduced form of the model allows an easy estimation and potential generalizations

Why a non parametric analysis?

- To avoid a priori assumptions on the relation between the team performance and its wage dispersion
- To highlight some non linearities, which could explain differences among previous results
- Because our endogeneous variable has a particular density



Data and variables

Variable	Nom	Définition	Min	Max	Moy	Ecart type
endogène:	$Perf\ E^D$	Performance de l'équipe qui joue à domicile, mesurée par la différence entre le nombre de points marqués et ceux encaissés.	-19	18	0.05	4.12
exogènes d'intérêt:	$Saltot^D$ $Saltot^E$	Masse salariale totale (en milliers) respectivement de l'équipe jouant à domicile et de l'équipe jouant à l'extérieur	1300	58630	11820	7619
	$ConcSal^D$ $ConcSal^E$	Respectivement la concentration des salaires au sein de l'équipe à domicile et de l'équipe à l'extérieur. On mesure cette concentration par l'index d'Herfindahl-Hirschmann ou par l'indice de Gini	HHI			
			0.12	0.64	0.20	0.057
			Gini			
			0.08	0.73	0.45	0.1
contrôle:	$R\ \hat{a}ge\ moy$	Ratio des âges moyens des joueurs des deux équipes	0.79	1.27	1.002	1

A two steps methodology

- First, a traditional linear approach

$$PerfE^D = \alpha + \beta_1 ConcSal^D + \beta_2 ConcSal^E$$
$$+\beta_3 Saltot^D + \beta_4 Saltot^E + \beta(X) + \varepsilon$$

- Then, the equivalent semi-parametric approach

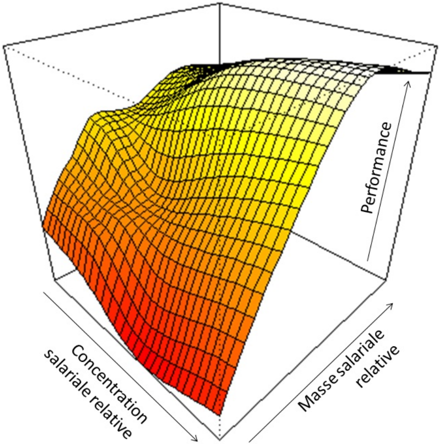
$$PerfE^D = \alpha + m\left(\frac{ConcSal^D}{ConcSal^E}, \frac{Saltot^D}{Saltot^E}\right) + \beta(X) + \varepsilon$$

The linear case

Forme	Variables	HHI		Gini	
		Coefficients	Erreur type	Coefficients	Erreur type
Linéaires	$Cst.$	2.310e+00***	7.753e-01	2.368e+0***	8.006e-01
	$Saltot^D$	3.052e-08***	8.789e-09	3.251e-08***	8.504e-09
	$Saltot^E$	-4.368e-08***	8.895e-09	-4.646e-0***	8.600e-09
	$ConcSal^D$	-2.480e+0***	8.979e-01	-1.338e+0***	5.021e-01
	$ConcSal^E$	2.392e+00***	9.011e-01	1.646e+0***	5.078e-01
	R âge moy	-2.041e+0***	7.411e-01	-2.248e+0***	7.524e-01
Nbre d'observations		9056		9056	

The semi-parametric case

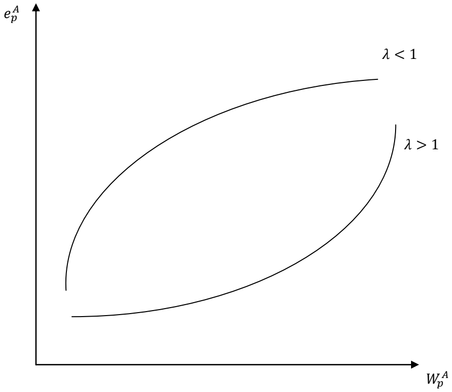
Variables	Gini	
	Coeff.	Erreur type
$Cst.$	•	•
$Saltot^D$	•	•
$Saltot^E$	•	•
$ConcSal^D$	•	•
$ConcSal^E$	•	•
R âge moy	-2.56***	0.825
$m\left(\frac{ConcSal^D}{ConcSal^E}, \frac{Saltot^D}{Saltot^E}\right)$	p-value = 7.99e-05 ***	



Conclusion

- Both competing theories have somehow a role to play
- The relation between wage dispersion and team performance has been proved to be clearly non linear
- The nature of the relation depends upon the relative total wage

Thank you..!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